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(54) Title: NOMINATIVE ADAPTIVE PROGRAMMING FOR A CLASS OF NON-LINEAR MIXED INTEGER PORTFOLIO OPTIMIZATION PROBLEMS
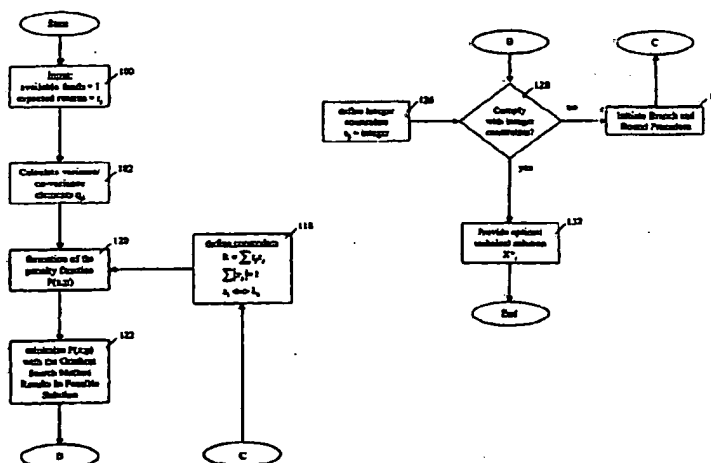
(57) Abstract: Apparatus and method for providing an optimal portfolio of assets by substantially maximizing the rate of return for a level of risk, or minimizing the level of risk for a rate of return. The apparatus and method incorporates a precise model formulation (100) whereby short sales are accounted for in the model specification. The apparatus and method also incorporates at the outset a precise model formulation whereby integer constraints (126) are accounted for in the model specification. Finally, the apparatus and method provides a recursive adaptation of the problem structure to unpredictable market conditions, by assuming that the model specification is deterministic rather than stochastic, and creating a directed feedback loop wherein nominative structural changes are put back into the model (130) and revalidated when market conditions deny the execution of the technical solution (132).

NOMINATIVE ADAPTIVE PROGRAMMING FOR A CLASS OF NON-LINEAR MIXED INTEGER PORTFOLIO OPTIMIZATION PROBLEMS

FIELD OF THE INVENTION

5   The present invention relates to a method and apparatus used to manage a portfolio of assets and, more specifically, to a method and apparatus for optimizing an executable portfolio of assets subject to non-linear, integer and sequential nominative constraints.

BACKGROUND OF THE INVENTION

10   Much has been written about the problem of selecting a portfolio of investments. The seminal work of Markowitz provides an analytical framework from which the problem can be cast into one that admits to formal methods of solution. *See* Markowitz, Harry M., "Portfolio Selection," Journal of Finance 7 (March 1952).

One simple rendition of the problem is to minimize the variability of the portfolio return (a proxy measure of the risk for the portfolio) such that the rate of return on the portfolio 15 is at least equal to some target amount, and that the total amount of the funds invested does not exceed the total amount of funds available for investment.

Two common assumptions underlie classical portfolio analysis. First, when efficient exchange is the rule rather than the exception, only so-called "long positions" are 20 addressed. Any investment in the portfolio must be greater than or equal to zero (a non-negative value). Short positions, where the investor borrows an asset and sells it with the intent of buying it back in order to deliver it when the loan is up, are not considered. In an ideal economic environment, the profit potential from short sales does not arise, because the market has already discounted these possibilities to be no more valuable than the cost of borrowing for the short

position.   Second, each investment alternative is treated as being infinitely divisible.   For example, one can invest in any fractional amount of any investment alternative, e.g., 10.357 units of asset j.

In addition to these two assumptions, traditional portfolio analysis does not consider the problem of execution risk.   Namely, the basic portfolio selection problem does not consider the inability to execute an indicated trade because the counter-party can no longer deliver the desired quantity or is not an acceptable counter-party at the level of business required or there are insufficient counter-parties in the market at the time.

While a powerful analytical device, the basic approach to portfolio selection loses much of its relevance, and even more of its simplicity, when applied to real world problems.   In reality, markets are not perfectly efficient and short positions are commonplace, not all assets are infinitely divisible and may have to be transacted in integer multiples of some given quantity and unpredictable counter-party limitations may require subjective modifications to the original problem specification.

SUMMARY

In one aspect, the present invention features techniques for providing a user with an optimal executable portfolio of assets subject to various constraints.

In general, the invention features a method for operating an electronic system for selecting an optimal executable portfolio of assets.   In one aspect of the invention one or more initial constraints are received, which may include, for example, a minimum portfolio rate of return, an upper or lower bound for an investment allocation, a requirement that all available funds should be exhausted in the portfolio of assets that may include short positions which in

turn requires that the absolute values of each investment allocation in the portfolio of assets adds

to one and the accommodation of integer constraints on one or more of the investment

alternatives in the portfolio. An optimal technical portfolio subject to these initial constraints is

calculated using a processor. Thereafter, one or more subjective constraints to modify the

5   optimal technical portfolio are received, which may include the elimination of one or more

investment alternatives, the modification or deletion of an existing constraint, or the addition of a

new constraint. Finally, an optimal market portfolio (optimal executable portfolio) subject to the

initial constraints and the subjective constraint(s) is calculated using the processor.

.   Preferred embodiments of the invention additionally feature calculating the

10   optimal technical portfolio by selecting investment allocations that substantially minimize a level

of risk of the portfolio captured as some measure of variability in the return on the portfolio of

assets for a given minimum portfolio rate of return. Calculating the optimal technical portfolio

may include defining a penalty function combining the risk of the portfolio of assets with the

initial constraints, and minimizing the penalty function using a non-linear solution method,

15   which may be a gradient search method. The penalty function may be redefined to account for

the satisfaction of one or more integer constraints. Thereafter, the optimal technical portfolio is

recalculated by minimizing the redefined penalty function using a non-linear solution method.

Integer programming methods such as the branch and bound method may be used to re-

determine the optimal technical portfolio such that the optimal technical portfolio complies with

20   the integer constraints. Calculating the optimal market portfolio may include redefining said

penalty function to account for one or more subjective constraints introduced subsequent to the

determination of the optimal technical portfolio, and minimizing the redefined penalty function

using a non-linear solution method. The method may further include calculating the optimal

3

technical portfolio and optimal market portfolio based on estimated rates of return and variances and co-variances of those returns among the available assets. The estimated rates of return and variances and co-variances of available assets may be obtained from a variety of sources and/or calculated in a number of different ways provided the units of measurement are the same across

5   the investment alternatives to ensure compatibility and comparability.

In a second aspect, the invention features a portfolio selection system having a processor connected to a memory or other storage device for storing asset information. The processor may connect to the memory or storage device through a network. A user interface connected to the processor receives one or more initial constraints, and the processor calculates

10   an optimal technical portfolio subject to the initial constraints. Thereafter, the user interface receives a subjective constraint to modify the optimal technical portfolio, and the processor calculates an optimal market portfolio subject to the initial constraints and the subjective constraint. The system may further include the additional features and the processor may perform the steps as described above.

15   In a third aspect, the system may include means for receiving one or more subjective constraints to modify the optimal technical portfolio, which may include the elimination of one or more investment alternatives, and/or the modification or deletion of an existing constraint, and/or the addition of a new constraint in order to derive an optimal market portfolio.

20   In general, in a fourth aspect, the invention features a computer readable medium having computer usable program code embodied therein. The program code may include code for performing the above-mentioned methods and procedures.

BRIEF DESCRIPTION OF THE DRAWINGS

Figs. 1 illustrates a flow chart of a system and method for finding an optimal technical portfolio of assets, where short selling is allowed and integer constraints exist for some of the portfolio assets;

5   Fig. 2 illustrates a flow chart of an adaptive programming system and method for finding an optimal market portfolio of assets, where the system and method of finding an optimal technical solution is modified to include sequential subjective modifications to the original portfolio selection problem.

10   **DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS**

I.   Introduction

In order to accommodate real world problems, a practical portfolio selection method should accommodate the following: (i) the viability of short positions, *e.g.*, selling
15   forward due to a belief that prices are going to fall at a higher rate than the cost of borrowing over the same period; (ii) the amount invested in any one asset may not be infinitely divisible, but rather, should be an integer multiple of some standard amount; and (iii) the trade execution of one or more of the investments in the solution set may not be possible for reasons that cannot be predicted or estimated. Such reasons may not be repeatable experiments and it may not be
20   possible to estimate their probabilities of occurrence. For example, the only available counter-party may no longer be a good delivery risk because it only became known after the determination of an optimal technical portfolio solution that a rogue trader surreptitiously bankrupted the trading firm.

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A.   Short Positions

In general, the inclusion of short sales serves to increase the universe of possible investment alternatives.   Moreover, these investment alternatives tend to have economic characteristics that are opposite those found in the long position investments, which can serve to

5   lower the risk of the entire portfolio.

From an accounting and regulatory perspective, one dollar of a long position and one dollar of a short position add up to two dollars of trading positions against which capital must be assigned. The capital requirement faced by the investor is indifferent as to the investor having a long or short position. Therefore, one approach for ensuring that the investment

10   allocations do not exceed capital limitations is to require that the absolute value of each of the allocations (expressed as a decimal fraction) add up to one.

In a technical solution context the non-negativity constraints on the investment allocations may be relaxed to allow for negative values. This relaxation may be done either across the board, or only for those investments that have negative expected returns (an expected

15   drop in price). The relaxation of the non-negativity condition for some of the asset allocations requires the absolute value constraint noted above. This new constraint ensures the allocation weights add up to one without the sum of any subset of the allocation weights exceeding one while allowing for negative values on some of the allocation weights. The absolute value constraint is a non-linear constraint that must be accommodated in a portfolio optimization

20   context.

B.   Integer Constraints

There are several market conditions that can lead to the invocation of integer constraints on the portfolio allocations. Many markets require that integer multiples of specified values be traded. For example the futures markets define the size of their contracts and fractions

5   cannot be traded. In the wholesale markets for foreign exchange and government securities integer multiples of specified face value amounts must be traded to receive standard wholesale brokerage commissions. In equities one often can obtain better pricing if round lots are traded. These conditions are common and must be accommodated. In practice such constraints often are enforced after portfolio optimization allocations have been derived by simple rounding and

10   without regard to re-balancing the optimal allocations. Such ad-hoc corrections result in sub-optimal allocations and can violate important constraints such as capital requirement limitations.

In order to ensure any such integer constraints are not violated integer-programming methods should be adopted to address this problem.

C.   Nominative Structural Changes

15   Market execution is unpredictable and unexpected new market information may result in the subjective revaluation of the amount of business one may do with any given counter-party. Moreover, these subjective revaluations may be different for different assets. These problems often arise subsequent to the derivation of an optimal technical solution and can deny the translation of such a solution into a market reality. One unfortunate aspect of such new

20   information is that they cannot be predicted and their probabilities cannot always be estimated if ever.

When such counter-party problems are encountered, they may be resolved through: (i) the elimination of one or more investment alternatives with corresponding

7

adjustments to the fund allocation and return constraints; and/or (ii) the addition, deletion or

modification of one or more new constraints to ensure the appropriate assets, and/or counter-

parties are restricted in the solution set to the correct amounts. These adaptations are nominative

since they are dependent in large part on non-quantifiable value assessments. In a portfolio

5   optimization context such changes must be fed back into the structure of the original problem so

as to re-derive an optimal technical solution that also admits to market implementation (i.e. is an

optimal market solution).

II.   A New Solution Method

        A.   Overview

10   Taken together, the modification of a basic portfolio optimization model to

accommodate the practical considerations noted above requires a new solution method. This

method should accommodate negative solution values, integer constraints and sequentially

applied post optimality nominative structural changes.

        The method presented below meets these objectives. It does so with a unique

15   blend of methodologies infused with guided but subjective (nominative) human interactions. At

its core, the method presented may be thought of as an adaptive multi-stage optimization of a

quadratic function that is subject to linear and non-linear deterministic constraints, allows for

negative solution values for the decision variables and is subject to sequentially applied

nominative structural changes. The method is divided into two stages. The first stage solves for

20   an optimal technical solution given the information available at the time the problem is first

specified. This stage is described in Fig. 1. The second stage solves for an optimal market

solution and is characterized by a succession of nominative structural adaptations to the first

8

stage problem specification necessary to achieve an optimal technical solution that also is a feasible market solution (i.e. an optimal market solution). This stage is described in Fig. 2.

B.   An "Executable Efficient Frontier"

5   It should be noted that in an economic context, the solution method presented is one that solves for a point on an "efficient frontier". By repeatedly generating these solutions, one for each different desired portfolio rate of return, the investor will generate a set of points, each of which consists of a desired rate of return and its corresponding minimum variance (or standard deviation) for the portfolio. A plot of these points with portfolio expected return on the

10   vertical axis (ordinate) and portfolio standard deviation on the horizontal axis (abscissa) yields an "efficient frontier". In keeping with the distinction between an optimal technical solution and an optimal market solution the economic notion of an efficient frontier would correspond to an efficient market frontier if said efficient frontier was executable at the time of its calculation or would be an efficient technical frontier if the optimal technical solutions could not be

15   implemented in the market.

C.   Measurement Of Returns, Their Variances And Co-Variances

Among the most important inputs to the model to be presented are the returns on the various assets considered for investment and their corresponding variances and co-variances.

20   In a sense, the model presented is independent of the measurement methods used for these data. That is not to say that different measurement methods may not yield different measurements or that different measurements will not lead to different optimal solutions. It is to say that the solution method developed herein can be applied to any set of asset returns, variances and co-

variances, regardless of how they are estimated, provided they are consistent as to method and units of measure.

For example, one can utilize a centered arithmetic daily average 6-month holding period rate of return for each asset and calculate the variances and co-variances among them over a several year period. Alternatively, one may use a Kalman filtering process to estimate the daily average holding period rates of return and their corresponding variances and co-variances. What is not recommended is the use of one method to measure the rates of return and another method to measure the variances and co-variances or applying one measurement method for some assets and another measurement method for other assets.

III. Optimal Technical Solution Methodologies (Figs. 1A and 1B)

A. Inputs and Variance/Co-Variance Calculation (Steps 100, 102)

Referring to Fig. 1A, the process begins at step 100 with the input of available funds I and the expected returns, r_j. The variance/co-variance elements are then calculated in step 102. The details of steps 100 and 102 are discussed further below.

B. Penalty Function Formulation (Step 120)

A penalty function formulation of the problem at hand simultaneously combines the objective of minimizing portfolio return variability subject to target return and budget constraints. The inclusion of the constraints is done in such a way that the solution of the objective function is penalized if any of the constraints are not met.

In the context of the portfolio problem at hand this new penalized function can be denoted:

$$P(x;p) = V(x) + p\Sigma(h_k(x))^2$$

where $V(x)$ is the original variance co-variance function, the $(h_k(x))^2$, for $k=1,...,K$, are

constraints whose second order term force the solution away from a minimum whenever $h(x)$

does not equal zero, and p is a non-zero positive scalar that serves the purpose of forcing $P(x;p)$

further away from a minimization if any of the constraints are not met.

5   The penalty function reformulation of the portfolio selection problem would be:

Minimize $P(x;p) = \Sigma\Sigma q_{ji}x_jx_i + p[(\Sigma r_jx_j - R)^2 + (\Sigma|x_j| - 1)^2]$

with respect to the x_j, for $j=1,...,n$, for successively smaller values of p.

Here, the objective is to minimize the variability of the portfolio return $\Sigma\Sigma q_{ji}x_jx_i$, subject to a

return constraint $\Sigma r_jx_j - R$, and possible short positions with the full utilization of available funds

10    $\Sigma|x_j| - 1$.

The minimization of the above function can be accomplished using non-linear

solution methods. One of such non-linear solution methods is the gradient search method.

C.   Gradient Search Method (Step 122)

15   The essence of the gradient search method is to move in the direction (gradient

vector when more than one variable is involved) of reducing the value of the new objective

function.

The gradient vector of the function $P(x;p)$ can be written:

$\nabla P(x;p) = (\delta P/\delta x_1, \delta P/\delta x_2, ..., \delta P/\delta x_n)$

20   where $\delta P/\delta x_j$ is the partial derivative of $P(x;p)$ with respect to x_j.

For $P(x:p)$ the expression for the gradient is:

$$\nabla P' = \begin{bmatrix} \partial P/\partial x_1 \\ \partial P/\partial x_2 \\ \partial P/\partial x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 + 0.4x_3 + 0.2x_2 + p2(\sum r_j x_j - R)(0.10) + p2(|\sum x_j| - 1)(\partial|x_1|/\partial x_1)_{x \neq 0} \\ 2x_2 + 1.0x_3 + 0.2x_1 + p2(\sum r_j x_j - R)(0.18) + p2(|\sum x_j| - 1)(\partial|x_2|/\partial x_2)_{x \neq 0} \\ 4x_3 + 1.0x_2 + 0.4x_1 + p2(\sum r_j x_j - R)(0.25) + p2(|\sum x_j| - 1)(\partial|x_3|/\partial x_3)_{x \neq 0} \end{bmatrix}$$

where $\nabla P = \nabla P(x;p)$, $\nabla P'$ is the transpose of ∇P, and $(\partial|x_j|/\partial x_j)_{x \neq 0}$ is the partial derivative of $|x_j|$

with respect to x_j, provided x_j does not equal zero, at which point the derivative is indeterminate.

As can be seen, the analytical form of the gradient components are non-linear. This occurs by virtue of the absolute value constraint. Since the relationships in the gradient are not linear, linear programming methods do not apply.

Theoretically the gradient can be evaluated continuously, but that is computationally prohibitive. Alternatively, one can move a fixed finite distance in the gradient direction. This operation can be expressed as moving from a given starting position $P(x;p)$, to the next position $P(x \pm t^*\nabla P(x;p);p)$, where t^* is the value of a single non-negative parameter ($t^* \geq 0$) that defines the fixed finite distance the variables are moved so as to optimize the function $P(x \pm t^*\nabla P(x;p);p)$, ($+t^*$ for a maximization and $-t^*$ for a minimization).

In the current minimization example, the optimal value of $t = t^*$ is determined by using a one-dimensional search procedure, or differential calculus, to find the value of $t = t^*$ that minimizes $P(x - t\nabla P(x;p);p)$. These calculations are repeated sequentially (iterations) until either $\nabla P(x;p) = 0$, and there is no further improvement in the objective function, or until $|\partial P/\partial x_j| \leq e$ for all $j=1,\ldots,n$, where e is an arbitrarily small error tolerance set by the user.

12

D.   Integer Programming

1.   Overview

In many allocation problems, the values of the decision variables (the amount of each individual investment) are acceptable only if they have integer values. For example, futures

5   contracts are traded in integer multiples of their contract size. Other examples would be trading U.S. government securities in integer multiples of $1 million, or integer multiples of 1000 shares of stock (a common "round lot" in equities). Such restrictions can be difficult to handle. Indeed, the solution methodology for an integer-programming problem is not singular. They vary depending in part on the precise characteristics of the problem. As a result of these "custom"

10   considerations, there are a bewildering number of methodological approaches to finding solutions for integer programming problems.

In general, there are two types of integer programming problems: (1) those where all the solution values must adhere to some form of integer constraint, *i.e.*, all integer problems; and (2) those for which only some values of the solution variables must have an integer value,

15   *i.e.*, mixed integer problems. Since mixed integer problems are more useful in real world applications, we focus the following discussion on such problems.

While no one computational procedure is best under all conditions, or is especially efficient for a mixed integer programming problem, the branch and bound method is considered to be among the most efficient. Before moving to this technique in detail, it is

20   important to first illustrate the character of integer constraints in the context of a portfolio optimization problem and to summarize the characteristics of some of the other alternative solution methods.

2.   Integer Constraints In Portfolio Optimization

For the portfolio optimization problem at hand, one solves for decimal fraction weights x_j, for $j=1,\ldots,n$, that minimize portfolio variance subject to specific constraints. As noted, some of the assets may have integer constraints associated with them. In the context of

5   the current discussion, such a constraint may be denoted x_jI = integer, for some j. That is, the product of the decimal fraction weight, x_j, times the total amount of investment funds available, I, must equal an integer value.

For example, suppose the solution value for asset 1 is $x_1 = 0.0820325$ and is the weight assigned to the asset after the application of the optimization methods noted above

10   without regard to integer constraints. Suppose asset 1 trades in integer multiples of $1 million. If the total amount of funds available for investment $I = \$100$ million, then $X_1I = \$8.20325$ million. Clearly this is not an integer multiple of $1 million. Accordingly one must now engage some form of integer programming method in order to obtain a solution that is an integer multiple of $1 million.

15

3.   Solution Methods

Four methods for solving integer constrained optimization problems are: (i) complete enumeration; (ii) rounding; (iii) cutting plane; and (iv) branch and bound. While the branch and bound method is used in this process each of the above alternatives is described

20   briefly with some comments on their strengths and weaknesses.

a.   Complete Enumeration

Complete enumeration, as its name implies, specifies every possibl solution to the problem and then selects the best solution, where ties are broken arbitrarily. In general this method is far too large an undertaking for any problem except those of the smallest size. For example, a problem with 10 variables, each with 10 possible outcomes, generates 10^{10} possible

5   outcomes for the entire set of variables.

b.   Rounding

Rounding is quick and simple, but typically violates other constraints and does not achieve an optimal integer solution. To illustrate the problem with rounding, suppose the decimal fraction solution for an investment alternative is, as illustrated above, $x_1 = 0.0820325$,

10   and the dollar amount invested is $8.20325 million (0.0820325 times $100 million). If one rounds up to $9 million, then the new value for the decimal fraction allocation for this investment would be $x_1 = 0.09$. Unfortunately, there is another constraint that also must be satisfied. Namely, one must allocate all of the total funds available, *i.e.*, the sum of the investment fractions must add up to 1. In the above example, this second condition has been

15   violated because the sum will now be 1.0079675. This is not a feasible solution, since it would require the investment of more funds than are available. A similar result occurs when one rounds down to 0.08. In that case, the sum of the allocation weights falls short by 0.0020325. The problems of rounding grow when the variable so constrained appears in several other constraints.

20   ### c.   Cutting Plane

The cutting plane method sequentially adds integer-satisfying constraints to an "original" non-integer constrained problem and solves each such newly constrained problem. If the new solution has an integer constraint violation on another variable, then the process is

repeated for this variable. This iterative sequence continues until there are no longer any integer constraint violations. In so doing, infeasible areas of the original solution space are "cut" away (fathomed), and the sequential solutions converge on the vertex that defines the optimal integer programming solution. Theoretically, it has been shown that this method will converge to an

5   optimal integer solution for an all integer problem. The primary shortcomings of the cutting plane method are twofold: (i) the possible lack of convergence to an optimal solution for a mixed integer problem (*e.g.*, integer constraints on some investment alternatives and a non-integer constraint on portfolio return in the current context); and (ii) it is computationally intensive.

d.   Branch and Bound

10   The branch and bound method uses separation, relaxation and fathoming to provide a solution that is not as computationally intensive as some other alternatives, and converges on an optimal solution for common types of mixed integer programming problems. Since the branch and bound method often avoids problems of convergence, and in many cases achieves a solution with greater efficiency than some of the other methods noted above, it is

15   adopted in this application.

While the branch and bound technique can be used to find optimal solutions, it also can be used to find a "nearly" optimal solution. Typically, this is done by terminating the procedure when the value of the objective function is within a pre-specified percentage (or numerical value) of the current least upper bound for the problem at hand. In other words, the

20   process may be terminated when the sequential solutions do not improve by some minimal increment.

E.   Penalty Functions and Integer Programming

In general, an integer programming problem is one that must satisfy integer constraints. One of the most common manifestations of an integer programming problem is a linear programming problem subject to such constraints set forth in step 126. Here, given the

5   integer constraint, the solution at any given stage is the solution to a linear programming problem. This is not the case for the portfolio optimization problem at hand, because the problem does not admit to linear programming solution methods.

As shown above, the inclusion of short sales transforms the original problem from one that can be solved using linear programming methods to one that is non-linear and must be

10   recast accordingly. Therefore, the quadratic programming problem was recast using a penalty function approach (step 120) and a gradient search method was used to solve the new problem (step 122). These changes do not lessen the relevance of integer programming methods including the use of Branch and Bound methods to accommodate integer constraints. Rather, it requires a solution hierarchy or sequencing that accommodates the needs of both methods. In the

15   event there are no integer constraints, there would be no need for the branch and bound algorithm. In this case, the penalty function/gradient search solution procedure noted above would find an optimum feasible solution without limitation as to the sign of the decision variables, but subject to the short sale constraint (sum of the absolute values).

Any applicable integer constraints are defined in step 126. The integer constraints

20   are validated against the initial feasible solution (step 128). If the initial feasible solution complies with the integer constraints, then the initial solution forms the optimal technical solution, denoted X^*_j, for j=1...n (step 132). If the initial solution does not comply with the

17

integer constraints, then the branch and bound procedure is initiated (step 130), until a solution is found that complies with the constraints.

Specifically, if there are integer constraints together with the possibility of short sales then the solution procedure would proceed in three steps. The first step would be to solve

5 the problem without regard to the integer constraint(s), but where one or more of the solution variables are allowed to be negative, arriving at the solution in step 122. The second step would be to redefine the penalty function P(x,p) to include the appropriate new constraint(s) intended to satisfy the integer constraint (branch) to be satisfied. For example the original penalty function was denoted:

10
$$\text{Min } P(x;p) = \Sigma\Sigma q_{ji}x_jx_i + p[(\Sigma r_jx_j - R)^2 + (\Sigma|x_j| - 1)^2].$$

If x_1 were subject to an integer constraint and the low branch of that constraint was $X_1 = 0.08$, then the new objective function would become:

$$\text{Min } P(x;p) = \Sigma\Sigma q_{ji}x_jx_i + p[(\Sigma r_jx_j - R)^2 + (\Sigma|x_j| - 1)^2 + (x_1 - 0.08)^2].$$

Additional such constraints would become additional quadratic terms in the penalty function

15 portion of the objective function (see step 118). The third step would·be to follow the branch and bound rules repeatedly to find an optimum feasible solution that satisfies the integer constraint(s). The solution for each sub-problem would be the solution to a problem of the type specified above. It should be noted that the branch and bound method does not require non-negativity on the part of the solution variables.

20
To illustrate the above concepts, consider the penalty function outlined above, with a constraint that the value of x_1 times ·100 must be an integer value, e.g., x_1 integer.

Step 1: Suppose the solution to a problem, without the integer constraint, produces a solution value for x_1 that does not admit to an integer solution, *i.e.*, $x_1 = 0.0820325$ (step 122).

Step 2: using the branch and bound technique (step 130), this solution becomes the first solution from which a separation of the non-integer solution takes place. Specifically two new problems replace the old problem. One of the two new problems (a branch) has the new constraint that $x_1 = 0.08$. The objective function to be solved for this problem, as redefined in step 120 according to the additional constraint, is the same as the example given above, namely:

$$\text{Min } P(x;p) = \Sigma\Sigma q_{ji}x_jx_i + p[(\Sigma r_jx_j - R)^2 + (\Sigma|x_j| - 1)^2 + (x_1 - 0.08)^2].$$

The other branch defines the other problem that has the constraint $x_1 = 0.09$. If the $X_1 = 0.09$ branch has the lowest value of the objective function and is a feasible solution, the $X_1 = 0.08$ branch is disregarded (fathomed). Since there are no other integer constraints to be satisfied, there is no reason for further separation (branching) and solving (bounding), and the solution with $x_1 = 0.09$ is the optimum solution (step 132).

F. Summary of The Optimal Technical Solution

The introduction of short sales led to the modification of the basic portfolio optimization problem from one with linear constraints to one with non-linear constraints. The solution to that modified problem required the use of gradient and penalty function methods.

The introduction of integer constraints led to a further modification of the problem to one that also required the use of integer programming methods for solution. Given the initial unconstrained (fully relaxed) solution to the problem, branch and bound methods

subsequently became sequential precursors to the non-linear solution methods required by the introduction of short sales.

Taken together, the existence of short sales and integer constraints results in a sequential non-linear process from which an optimal technical solution is derived.

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IV.   Nominative Adaptive Programming: An Optimal Market Solution

A.   Overview

The problem specification to this point has not dealt with the notion of random variables and has not needed to. That is to say, the problem has not had to address the

10   probability of going from one state to another state depending on which action or decision is taken. The integer constraints are known with certainty and they are either satisfied or not by virtue of the mathematical structure of the problem. The integer constraints are not violated or met because of a random event at this stage of the solution.

Where random events do enter the solution is in the application of the technical

15   solution to the market. Here, as noted at the outset, random events may deny the implementation of the technical solution. Moreover, these unanticipated market events may require structural changes to the problem such as the addition of new constraints, the deletion of old constraints, changes in existing constraints and combinations of the above.

One approach to solving problems with random events is Stochastic Dynamic

20   Programming. This approach requires that the probabilities of alternative outcomes triggered by earlier actions are known or are estimable. Another approach is Nominative Adaptive Programming. This method enables one to accommodate random events, with unknown and

non-estimabl probabilities, and the nominative structural changes they introduce.   Each

approach is described below.

    B.   **Stochastic Dynamic Programming**

5   Given the above probabilities and possible structural changes, the investor's

objective can be re-stated as trying to minimize the expected value of $P^*(x;p)$ at any stage over

all the possible structural changes ξ given their known or estimable probabilities.   The

probabilities for such a solution method must be known or estimable for the class of problems at

hand. This is not the case for reasons spelled out below.

10   The place where uncertainty arises is in the implementation of the optimal

technical solution.   Here, the market may not always oblige the investor in achieving the

technical solution recommended. Unfortunately, execution risk is not one for which accurate (if

any) probabilities are known or estimable. Such estimates depend on information that is not

readily available. For example, not all counter-parties are equal in their ability to execute trades

15 and some may not be adequate. Moreover, the amount of capital available at any one firm is not

known to the market place on a real time trading basis (if on a daily basis). Furthermore, failures

to deliver can emerge from correspondent relationships that are several steps removed from the

trading at hand and not observable.   In addition, failures are not well documented and

management practices can change as quickly as a rouge trader can unexpectedly bankrupt a firm.

20 For these reasons, the required probabilities are neither known nor estimable and stochastic

dynamic programming is not applicable to the class of problems being considered.

C.   Nominative Adaptive Programming

Nominative adaptive programming is the process by which the structure of an original problem is repeatedly modified and resolved subsequent to an optimal technical solution until an optimal technical solution is found that also can be implemented in the market.

5   Owing to unexpected market conditions, post optimality changes in the technical specification may be necessary. These changes are not estimable before the fact. Rather they are events that trigger subsequent changes to the problem specification that are nominative. That is to say, the changes made by the investor are subjective and must be fed back into the problem structure to accommodate the execution constraints the market has unexpectedly imposed.

10   Examples of such unpredictable market events include, but are not limited to, changes in: (i) demand for or availability of one or more assets of reference; (ii) credit worthiness of one or more counter-parties; (iii) legal requirements regarding portfolio composition; (iv) tax treatment for some of the assets; (v) payment terms; (vi) management of the organization behind the asset; (vii) earnings from unexpected changes in materials costs; (viii) expected litigation outcomes; (ix) foreign exchange exposure; (x) general market conditions (natural catastrophes); and (xi) social structure and management preferences.

When such events occur the investor has to adjust the optimal technical solution to accommodate the market reality. These modifications take the form of changes to the structure of the problem such as: (i) the elimination of one or more investment alternatives; (ii) the modification of an existing constraint; (iii) the addition of new constraints; (iv) the deletion of one or more old constraints; and (v) combinations of the above. While the general nature of such a structural change may be known and is "objective", e.g., do less business with a counter-party whose credit worthiness has eroded, the precise character (add a new constraint or change

22

an old one) and exact amount of such a change may be a subjective decision. When subjective

changes are made to the structure of the problem, a new optimal technical solution must found to

ensure that the minimum variance objective is being met, as are the various old and new

constraints.

5   Referring to Figs. 2A and 2B, the methodology of adaptive programming consists

of two major phases. The first phase is the technical solution methodology presented in the

previous sections, resulting in the optimal technical solution of step 132. (Thus, steps 100, 102,

120, 122, 128, 126, 130 and 118 are the same in Figs. 1 and 2.) The second phase consists of

two steps and a sequential application of the two phases of the solution algorithm. The two steps

10   in the second phase are: (i) the market validity check of the technical solution (step 134); and (ii)

the nominative adjustments to the structure of the problem (step 136). The sequential application

of these steps involves feeding the structural changes back into the technical solution

methodology. Specifically, the structural changes are fed back into the system as constraints, in

step 118. The two step adaptive modifications are repeated until an optimal technical solution is

15   found that is also a feasible market solution. Such solution is provided as the optimal market

solution, denoted $X*_j$, for j=1...n (step 138).

          A consequence of making subjective structural changes to the problem is the

possibility that a subsequent optimal technical solution may not be feasible. In this case the

investor must change or make additional modifications so as to eliminate the previous offending

20   changes. These corrective changes also are subjective decisions, albeit ones that are directed by

the identification of the offending structural changes made previously. Excluding logical

inconsistencies, e.g., x_1 must be less than 0.08 and greater than 0.09, it may be that the market

does not admit to a feasible solution for reasons that cannot be addressed through further

modifications of the allocation constraints. Here, the only further change that may achieve a technical and market optimal solution may be a reduction in the desired rate of return.

Regardless of the nature of the infeasibilities encountered, the corrections (save for logical errors) are subjective and are introduced through the same feedback loop that defines

5   the adaptive programming algorithm.

V.   Implementation

Fig. 3 is a block diagram of one embodiment of a portfolio optimization system described herein. Central processing unit ("CPU") 200 connects to storage device 202 that stores

10   the necessary information for generating an optimized portfolio of assets. Such information may include an asset information database 204, an asset correlation database 206, system constraints 208 and user constraints 210. Storage device 202 may be any device capable of storing and retrieving information, such as a hard disk drive or random access memory ("RAM"). In alternative embodiments, the information stored on storage device 202 may be stored on separate

15   storage devices, directly or indirectly coupled to CPU 200.

Asset price source 212 connects to CPU 200 for providing price information for various assets to the optimization system. The price information may include current and past price information. In an alternative embodiment, storage device 202 stores historical price inputs for the assets, and asset price source 212 provides asynchronous price updates to the

20   optimization system.

CPU 200 executes various software routines for the optimization system in memory 214. The software routines may include updating procedure 216, user interface

24

procedure 218 and optimization procedure 220. In an alternative embodiment, separate CPU's

may execute the software routines on different computers.

A user may communicate with the optimization system through input/output

device 222 connected to CPU 200. Input/output device 222 may be a keyboard, mouse and

5   terminal of a computer performing the optimization method. In an alternative embodiment,

input/output device 222 may be a computer or other device communicating with CPU 200

through a network such as a LAN, WAN, or the Internet.

The optimization system receives historical price inputs from asset price source

212. The historical price inputs undergo normalization, if necessary, for providing a common

10   unit of measure. Updating procedure 216 generates asset information from the historical price

inputs, including asset information database 204 and asset correlation database 206. Updating

procedure 216 stores the asset information on storage device 202. Asset information database

204 stores information pertaining to the available assets, such as their rates of return, r_j. Asset

correlation database 206 stores the variance co-variance elements, q_{ji}.

15   Numerous methods exist for predicting the rate of return r_j and level of risk for the

available assets. The information may be obtained from an external data source, derived from

the price history of the assets, set by the user, or a combination of the above. For example, the

optimization system could use the average rate of return for each asset over a specific period of

time as the expected rate of return. Optionally, the user could modify some of those values

20   based on other external factors, such as higher or lower expected earnings for a company in a

particular period. The units of measure for the assets should be normalized, if necessary. For

example, the value of all assets could be converted to U.S. dollars as a common unit of

measurement.

User interface proc dure 218 may perform preprocessing to set up initial

constraints that the optimization system will place on the portfolio, as defined in step 118. The

initial constraints may comprise system constraints 208 and/or user constraints 210. A user

interacting with user interface procedure 218 through input/output device 222 may define new

5   constraints or modify previously saved constraints. System constraints 208 may be applicable to

every user of the optimization system. Optionally, access to system constraints 208 may be

limited to a defined set of users, such as a system administrator. User constraints 210 may be set

and modified by an individual user of the optimization system.

Thereafter, the asset information and the initial constraints are submitted to

10   optimization procedure 220, which generates a recommended portfolio using an iterative

process, as described herein. Optimization procedure 220 uses information stored in asset

information database 204 and asset correlation database 206 to calculate an optimized portfolio

subject to any system constraints 208 and user constraints 210. User interface procedure 218

displays the recommended portfolio to a user through input/output device 222, and accepts

15   additional constraints as directed nominative structural changes, in step 136.

If an additional constraint is imposed, which could include removing a previously

imposed constraint, the additional constraint is submitted to optimization procedure 220 for re-

generating a recommended portfolio subject to the additional constraints. The recommended

portfolio is used as a starting point for the iterative process of optimization procedure 220. The

20   process is repeated until there are no additional constraints, at which point the final optimized

portfolio is displayed to the user through input/output device 222 (step 138).

The portfolio optimization process may be repeated in real time as asset price

source 212 provides asynchronous price updates to the optimization system. When the

26

optimization system receives price updates, updating procedure 216 updates the asset

information database 204 and asset correlation database 206. The optimization process may be

repeated as new data are received, or on a periodic basis. At the option of the user, any existing

constraints stored on storage device 202 may be shown the user, thereby permitting modification

5   or elimination of the existing constraints, or the addition of new constraints. Updating procedure

216 uses asynchronous price updates to update the asset information stored on asset information

database 204, which is then used to calculate an updated portfolio. A previously recommended

portfolio may be used as a starting point for the iterative optimization process, to quicken the

calculations.

10

VI.   Conclusions

        The output of a properly structured mathematical model of portfolio selection will

provide the analyst with a technical solution. Such solutions typically do not address both short

sales and integer constraints on the decision variables. When such conditions are encountered

15   the technical solution is subject to ad-hoc modifications to accommodate them. Moreover, when

the solution is to be implemented in the market place unpredictable impediments to execution

may come into play. When these market restrictions surface the proposed solution is further

twisted and turned to conform to the reality of the marketplace with no effort to re-balance the

model or measure the effects of these ad hoc changes to the solution.

20   The preferred embodiment discussed herein addresses the practical problems of

portfolio selection in at least four ways. First, it provides for the minimization of portfolio risk

subject to a constraint on the portfolio rate of return. Second, it incorporates at the outset a

precise model formulation whereby short sales are accounted for in the model specification.

27

Third, it incorporates at the outset a precise model formulation whereby integer constraints are accounted for in the model specification. Finally, it provides a recursive adaptation of the problem structure to unpredictable market conditions. This adaptive process has two basic components: (i) it assumes a priori that the model specification is deterministic rather than stochastic as the dynamics of the market place do not lend themselves to any probabilistic estimation in either an expected value or Bayesian sense; and (ii) it creates a directed feedback loop wherein nominative structural changes are put back into the model and revalidated when market conditions deny the execution of the technical solution. The process becomes a recursive one as the assemblage of structural modifications may at times lead to infeasible technical or market solutions that require further adaptations.

The adaptive process presented bridges the gap between the abstraction of a mathematical model and the reality of the marketplace. The method presented incorporates nominative decisions within the boundaries of a formal process. In so doing, the methodology presented provides a more robust solution to the problem of portfolio selection than conventional approaches that do not introduce subjective modifications to the model structure in a formal way.

It should be understood that the above description is only representative of illustrative embodiments. For the convenience of the reader, the above description has focused on a representative sample of all possible embodiments, a sample that teaches the principles of the invention. The description has not attempted to exhaustively enumerate all possible variations. That alternative embodiments may not have been presented for a specific portion of the invention, or that further alternative embodiments may be available for only a portion of the above-described embodiments, is not to be considered a disclaimer of those alternative embodiments. One of ordinary skill will appreciate that many alternative embodiments that have

not been specifically enumerated are within the literal scope of the following claims, and that

others are equivalent.

CLAIMS

1.   A method for operating an electronic system for selecting a portfolio of assets from a plurality of assets, comprising:

receiving one or more initial constraints;

calculating an optimal technical portfolio subject to said initial constraints using a processor;

receiving a subjective constraint to modify said optimal technical portfolio; and

calculating an optimal market portfolio subject to said initial constraints and said subjective constraint using said processor.

2.   The method according to claim 1 wherein said initial constraints comprise a minimum target portfolio rate of return.

3.   The method according to claim 2 wherein calculating said optimal technical portfolio comprises selecting investment allocations that substantially minimize a level of risk of said portfolio for said minimum portfolio rate of return.

4.   The method according to claim 1 wherein said initial constraints comprise an upper or lower bound for an investment allocation.

5.   The method according to claim 1 wherein said initial constraints comprise a requirement that all available funds should be exhausted in said portfolio of assets.

30

6.   The method according to claim 5 wherein said initial constraint comprises a requirement that each of a plurality of solution values of each investment allocation in said portfolio of assets take on positive or negative values.

7.   The method according to claim 5 wherein said initial constraint comprises a requirement that the sum of the absolute values of each investment allocation in said portfolio of assets adds to one.

8.   The method according to claim 1 wherein calculating said optimal technical portfolio comprises utilizing a penalty function combining the risk of said portfolio of assets with said initial constraints.

9.   The method according to claim 8 wherein calculating said optimal technical portfolio further comprises minimizing said penalty function using a non-linear solution method.

10.   The method according to claim 9 wherein said non-linear solution method is a gradient search method.

11.   The method according to claim 1 further comprising redefining said initial constraint to account for one or more integer constraints.

12.   The method according to claim 11 further comprising recalculating said optimal technical portfolio by minimizing said penalty function.

13.   The method according to claim 9 further comprising using an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints.

14.   The method according to claim 13 wherein said integer programming method is the branch and bound method.

15.   The method according to claim 9 wherein calculating said optimal market portfolio comprises redefining said penalty function to account for said subjective constraint.

16.   The method according to claim 15 wherein calculating said optimal market portfolio further comprises minimizing said redefined penalty function using a non-linear solution method.

17.   The method according to claim 1 further comprising seeking said optimal technical portfolio utilizing recursive features embodied in a dynamic programming approach.

18.   The method according to claim 1 wherein one type of said subjective constraint is the elimination of one or more investment alternatives.

19.   The method according to claim 1 wherein one type of said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

20.   The method according to claim 1 further comprising calculating said optimal technical portfolio based on estimated rates of return of available assets and their respective variances and co-variances.

21.   The method according to claim 20 further comprising calculating said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

22.   The method according to claim 20 further comprising calculating said estimated rates of return and variances and co-variances of available assets using a Kalman filtering process.

23.   A portfolio selection system comprising:

a memory that stores asset information;

a processor connected to said memory;

a user interface connected to said processor for receiving one or more initial constraints;

wherein said processor calculates an optimal technical portfolio subject to said initial constraint;

wherein said user interface receives a subjective constraint to modify said optimal technical portfolio; and

wherein said processor calculates an optimal market portfolio subject to said initial constraints and said subjective constraint.

24.   The system according to claim 23 wherein said initial constraints comprise a minimum target portfolio rate of return.

25.   The system according to claim 24 wherein said processor calculates said optimal technical portfolio by selecting investment allocations that substantially minimize a level of risk of said portfolio for said minimum portfolio rate of return.

26.   The system according to claim 23 wherein said initial constraints comprise an upper or lower bound for an investment allocation.

27.   The system according to claim 23 wherein said initial constraints comprise a requirement that all available funds should be exhausted in said portfolio of assets.

28.   The system according to claim 27 wherein said initial constraint comprises a requirement that the solution values of each investment allocation in said portfolio of assets take on positive or negative values.

29.   The system according to claim 27 wherein said initial constraint comprises a requirement that the sum of the absolute values of the investment allocation in said portfolio of assets adds to one.

30.   The system according to claim 23 wherein said processor utilizes a penalty function combining the risk of said portfolio of assets with said initial constraints.

31.   The system according to claim 30 wherein said processor calculates said optimal technical portfolio by minimizing said penalty function using a non-linear solution method.

32.   The system according to claim 31 wherein said non-linear solution method is a gradient search method.

33.   The system according to claim 31 wherein said processor redefines said initial constraints to account for one or more integer constraints.

34.   The system according to claim 33 wherein said processor recalculates said optimal technical portfolio by minimizing said penalty function using a non-linear solution method.

35.   The system according to claim 31 wherein said processor uses an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints.

36.   The system according to claim 35 wherein said integer programming method is the branch and bound method.

37.   The system according to claim 31 wherein said processor redefines said penalty function to account for said subjective constraint.

38.   The system according to claim 37 wherein said processor calculates said optimal market portfolio by minimizing said redefined penalty function using a non-linear solution method.

39.   The system according to claim 23 wherein said processor calculates said optimal technical portfolio utilizing recursive features embodied in a dynamic programming approach.

40.   The system according to claim 23 wherein said subjective constraint is the elimination of one or more investment alternatives.

41.   The system according to claim 23 wherein said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

42.   The system according to claim 23 wherein said processor calculates said optimal technical portfolio based on estimated rates of return of available assets and their respective variances and co-variances.

43.   The system according to claim 42 wherein said processor calculates said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

44.   The system according to claim 42 wherein said processor calculates said estimated rates of return and variances and co-variances of available assets potentially using a Kalman filtering process.

45.   A method for operating an electronic system for selecting a portfolio of assets from a plurality of assets, comprising:

receiving one or more initial constraints;

defining a penalty function combining the risk of said portfolio of assets with said initial constraints;

calculating an optimal technical portfolio subject to said initial constraints using a processor by minimizing said penalty function using a non-linear solution method;

using an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints;

receiving a subjective constraint to modify said optimal technical portfolio;

37

redefining said penalty function to account for said subjective constraint; and

calculating an optimal market portfolio using said processor by minimizing said redefined penalty function using said non-linear solution method.

46.   The method according to claim 45 wherein said initial constraints comprise a minimum target portfolio rate of return.

47.   The method according to claim 45 wherein said initial constraints comprise an upper or lower bound for an investment allocation.

48.   The method according to claim 45 wherein said initial constraints comprise a requirement that all available funds should be exhausted in said portfolio of assets.

49.   The method according to claim 48 wherein said initial constraints comprise a requirement that solution values of each investment allocation in said portfolio of assets take on positive or negative values.

50.   The method according to claim 48 wherein said initial constraints comprise a requirement that the sum of the absolute values of the investment allocation in said portfolio of assets adds to one.

51.   The method according to claim 45 wherein said non-linear solution method is a gradient search method.

52. The method according to claim 45 wherein said integer programming method is the branch and bound method.

53. The method according to claim. 45 further comprising calculating said optimal technical portfolio utilizing the recursive features embodied in a dynamic programming approach.

54. The method according to claim 45 wherein said subjective constraint is the elimination of one or more investment alternatives.

55. The method according to claim 45 wherein said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

56. The method according to claim 45 further comprising calculating said optimal technical portfolio based on estimated rates of return and variances and co-variances of available assets.

57. The method according to claim 56 further comprising calculating said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

58.   The method according to claim 56 further comprising calculating said estimated rates of return and variances and co-variances of available assets using a Kalman filtering process.

59.   A portfolio selection system comprising:

a memory that stores asset information;

a processor connected to said memory;

a user interface connected to said processor for receiving one or more initial constraints;

wherein said processor defines a penalty function combining the risk of said portfolio of assets with said initial constraints;

wherein said processor calculates an optimal technical portfolio subject to said initial constraints by minimizing said penalty function using a non-linear solution method;

wherein said processor uses an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints;

wherein said user interface receives a subjective constraint to modify said optimal technical portfolio;

wherein said processor redefines said penalty function to account for said subjective constraint; and

wherein said processor calculates an optimal market portfolio by minimizing said redefined penalty function using said non-linear solution method.

60.   The system according to claim 59 wherein said initial constraints comprise a minimum target portfolio rate of return.

61.   The system according to claim 59 wherein said initial constraints comprise an upper or lower bound for an investment allocation.

62.   The system according to claim 59 wherein said initial constraints comprise a requirement that all available funds should be exhausted in said portfolio of assets.

63.   The system according to claim 62 wherein said initial constraints comprise a requirement that the solution values of each investment allocation in said portfolio of assets take on positive or negative values.

64.   The system according to claim 62 wherein said initial constraints comprise a requirement that the sum of the absolute values of the investment allocation in said portfolio of assets adds to one.

65.   The system according to claim 59 wherein said non-linear solution method is a gradient search method.

66.   The system according to claim 59 wherein said integer programming method is the branch and bound method.

67.   The system according to claim 59 wherein said processor calculates said optimal technical portfolio utilizing the recursive features embodied in the dynamic programming approach.

68.   The system according to claim 59 wherein said subjective constraint is the elimination of one or more investment alternatives.

69.   The system according to claim 59 wherein said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

70.   The system according to claim 59 wherein said processor calculates said optimal technical portfolio based on estimated rates of return of available assets and their respective variances and co-variances.

71.   The system according to claim 70 wherein said processor calculates said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

72.   The system according to claim 70 wherein said processor calculates said estimated rates of return and variances and co-variances of available assets potentially using a Kalman filtering process.

73.   A portfolio selection system comprising:

42.

means for receiving one or more initial constraints;

means for calculating an optimal technical portfolio subject to said initial constraints using a processor;

means for receiving a subjective constraint to modify said optimal technical portfolio; and

means for calculating an optimal market portfolio subject to said initial constraints and said subjective constraint using said processor.

74.   The system according to claim 73 wherein said initial constraints comprise a minimum target portfolio rate of return.

75.   The system according to claim 74 wherein calculating said optimal technical portfolio comprises selecting investment allocations that substantially minimize a level of risk of said portfolio for said minimum portfolio rate of return.

76.   The system according to claim 73 wherein said initial constraints comprise an upper or lower bound for an investment allocation.

77.   The system according to claim 73 wherein said initial constraints comprise a requirement that all available funds should be exhausted in said portfolio of assets.

78.   The system according to claim 77 wherein said initial constraints comprise a requirement that the solution values of each investment allocation in said portfolio of assets take on positive or negative values.

79.   The system according to claim 77 wherein said initial constraints comprise a requirement that the sum of the absolute values of the investment allocation in said portfolio of assets adds to one.

80.   The system according to claim 73 wherein calculating said optimal technical portfolio comprises means for defining a penalty function combining the risk of said portfolio of assets with said initial constraints.

81.   The system according to claim 80 wherein calculating said optimal technical portfolio further comprises means for minimizing said penalty function using a non-linear solution method.

82.   The system according to claim 81 wherein said non-linear solution method is a gradient search method.

83.   The system according to claim 81 further comprising means for redefining said penalty function to account for all specified integer constraints.

84. The system according to claim 83 further comprising means for recalculating said optimal technical portfolio by minimizing said redefined penalty function using a non-linear solution method.

85. The system according to claim 81 further comprising means for using an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints.

86. The system according to claim 85 wherein said integer programming method is the branch and bound method.

87. The system according to claim 81 wherein calculating said optimal market portfolio comprises means for redefining said penalty function to account for said subjective constraints.

88. The system according to claim 87 wherein calculating said optimal market portfolio further comprises means for minimizing said redefined penalty function using a non-linear solution method.

89. The system according to claim 73 further comprising means for calculating said optimal technical portfolio utilizing the recursive features embodied in the dynamic programming approach.

90.   The system according to claim 73 wherein said subjective constraint is the elimination of one or more investment alternatives.

91.   The system according to claim 73 wherein said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

92.   The system according to claim 73 further comprising means for calculating said optimal technical portfolio based on estimated rates of return and variances and co-variances of available assets.

93.   The system according to claim 92 further comprising means for calculating said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

94.   The system according to claim 92 further comprising means for calculating said estimated rates of return and variances and co-variances of available assets using a Kalman filtering process.

95.   An article of manufacture comprising:

a computer readable medium having computer usable program code embodied therein, said computer usable program code containing executable instructions that, when executed, cause a computer to perform the steps of:

receiving one or more initial constraints;

calculating an optimal technical portfolio subject to said initial constraints using a

processor;

receiving a subjective constraint to modify said optimal technical portfolio; and

calculating an optimal market portfolio subject to said initial constraint and said

subjective constraints using said processor.

96.   The article according to claim 95 wherein said initial constraints comprise

a minimum target portfolio rate of return.

97.   The article according to claim 96 wherein calculating said optimal

technical portfolio comprises selecting investment allocations that substantially minimize a level

of risk of said portfolio for said minimum portfolio rate of return.

98.   The article according to claim 95 wherein said initial constraints comprise

an upper or lower bound for an investment allocation.

99.   The article according to claim 95 wherein said initial constraints comprise

a requirement that all available funds should be exhausted in said portfolio of assets.

100.   The article according to claim 99 wherein said initial constraints comprise

a requirement that the solution values of each investment allocation in said portfolio of assets

take on positive or negative values.

101.   The article according to claim 99 wherein said initial constraints comprise a requirement that the sum of the absolute values of the investment allocation in said portfolio of assets adds to one.

102.   The article according to claim 95 wherein calculating said optimal technical portfolio comprises defining a penalty function combining the risk of said portfolio of assets with said initial constraints.

103.   The article according to claim 102 wherein calculating said optimal technical portfolio further comprises minimizing said penalty function using a non-linear solution method.

104.   The article according to claim 103 wherein said non-linear solution method is a gradient search method.

105.   The article according to claim 103 further comprising redefining said penalty function to account for all specified integer constraints.

106.   The article according to claim 105 further comprising recalculating said optimal technical portfolio by minimizing said redefined penalty function using a non-linear solution method.

107. The article according to claim 103 further comprising using an integer programming method to redetermine said optimal technical portfolio such that said optimal technical portfolio complies with all specified integer constraints.

108. The article according to claim 107 wherein said integer programming method is the branch and bound method.

109. The article according to claim 103 wherein calculating said optimal market portfolio comprises redefining said penalty function to account for said subjective constraint.

110. The article according to claim 109 wherein calculating said optimal market portfolio further comprises minimizing said redefined penalty function using a non-linear solution method.

111. The article according to claim 95 further comprising calculating said optimal technical portfolio utilizing recursive features embodied in a dynamic programming approach.

112. The article according to claim 95 wherein said subjective constraint is the elimination of one or more investment alternatives.

113.   The article according to claim 95 wh rein said subjective constraint is the modification or deletion of an existing constraint, or the addition of a new constraint.

114.   The article according to claim 95 further comprising calculating said optimal technical portfolio based on estimated rates of return and variances and co-variances of available assets.

115.   The article according to claim 114 further comprising calculating said estimated rates of return and variances and co-variances of available assets based on a centered arithmetic daily average rate of return over a period of time.

116.   The article according to claim 114 further comprising calculating said estimated rates of return and variances and co-variances of available assets using a Kalman filtering process.
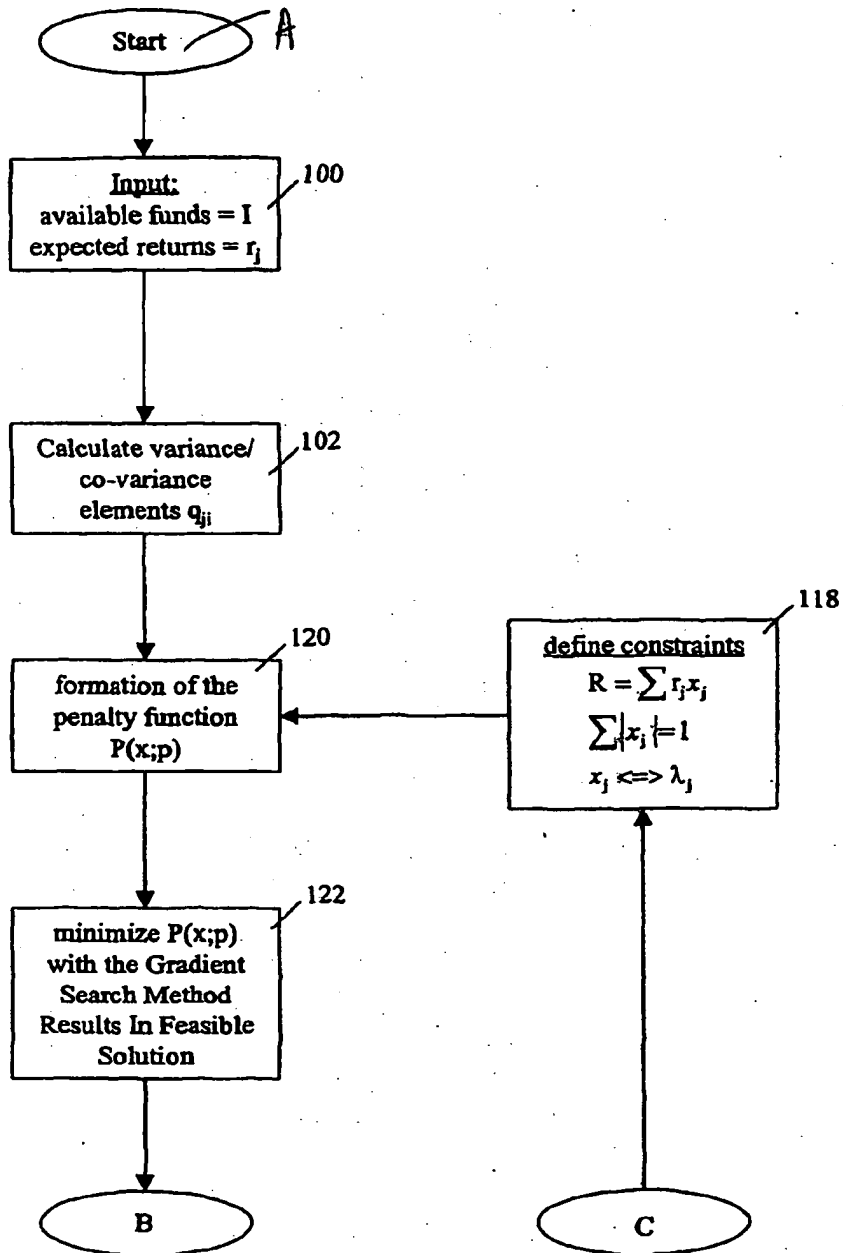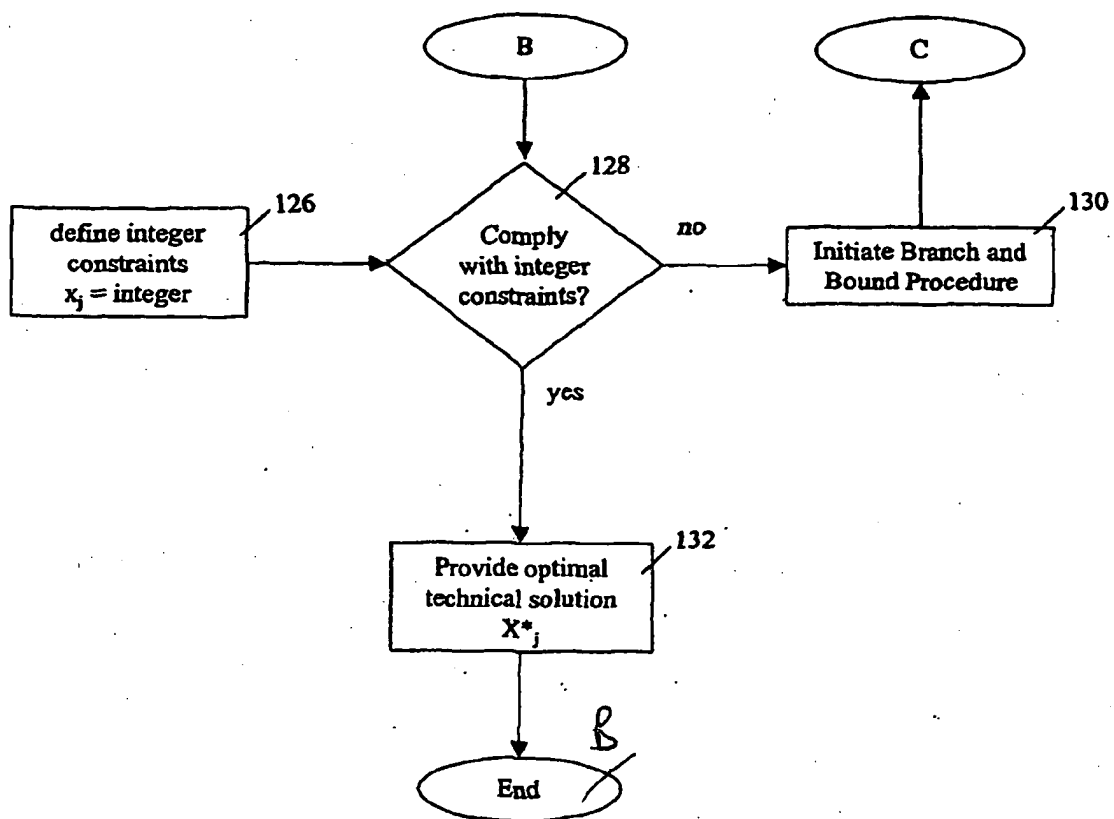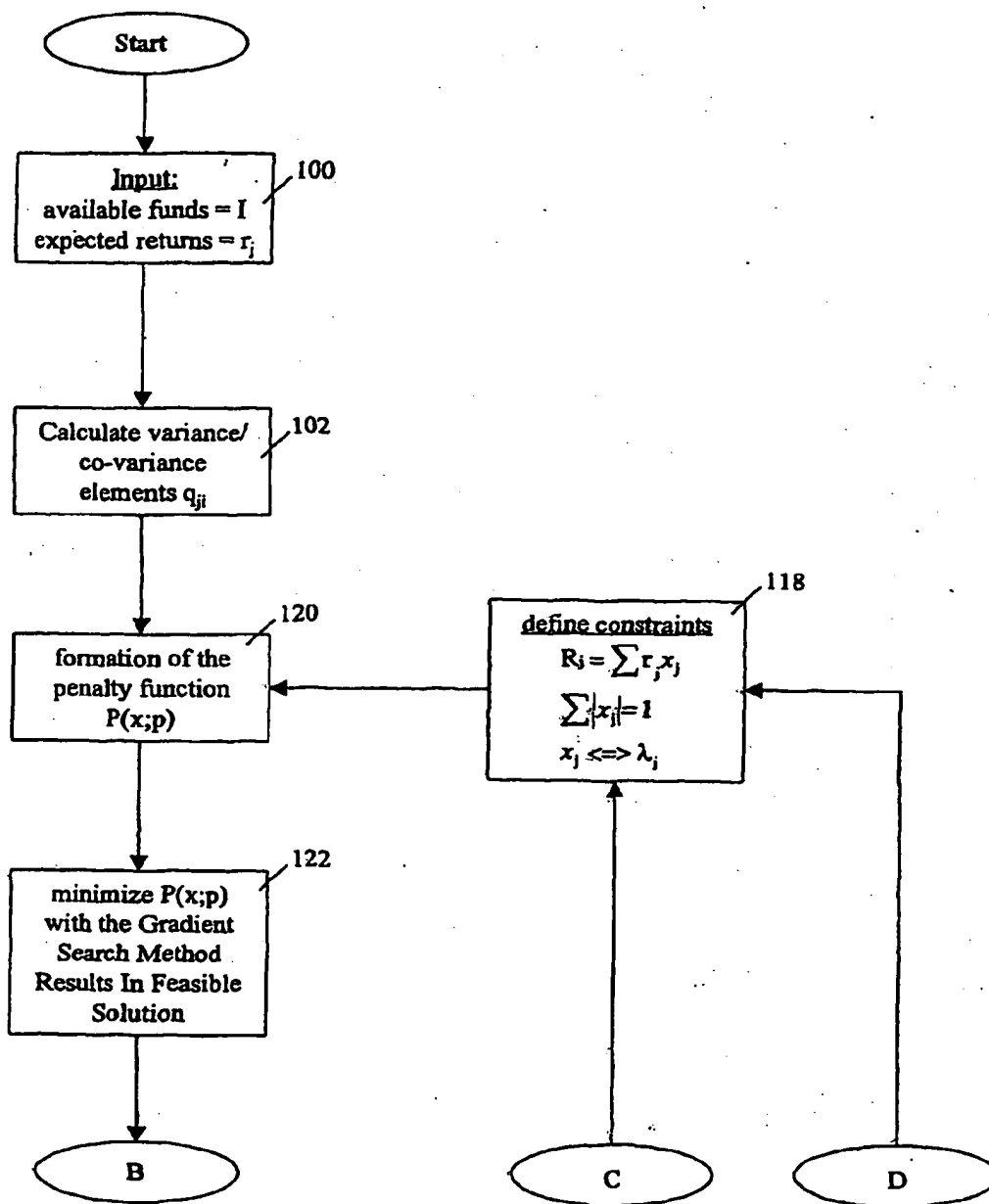
Fig. 1A

Fig. 1B

```
                    ┌──────────┐
                    │  Start   │
                    └────┬─────┘
                         │
                         ▼
              ┌─────────────────────┐  100
              │      Input:         │
              │ available funds = I │
              │ expected returns = rⱼ│
              └──────────┬──────────┘
```

$$\text{available funds} = I$$
$$\text{expected returns} = r_j$$

Calculate variance/co-variance elements q_{ji} — 102

formation of the penalty function $P(x;p)$ — 120

define constraints — 118

$$R_j = \sum r_j x_j$$
$$\sum |x_j| = 1$$
$$x_j \lessgtr \lambda_j$$

minimize $P(x;p)$ with the Gradient Search Method Results In Feasible Solution — 122
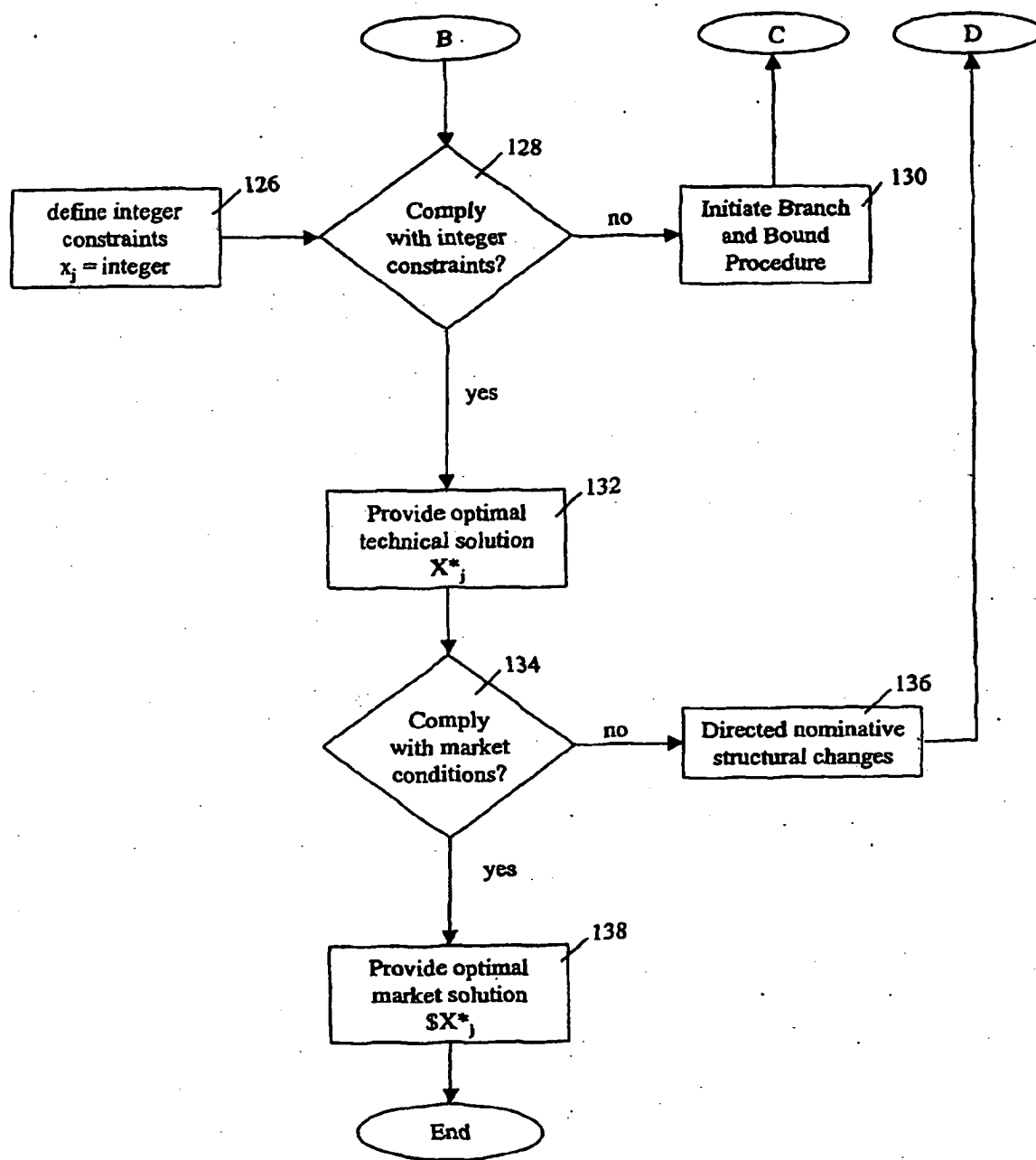
B   C   D
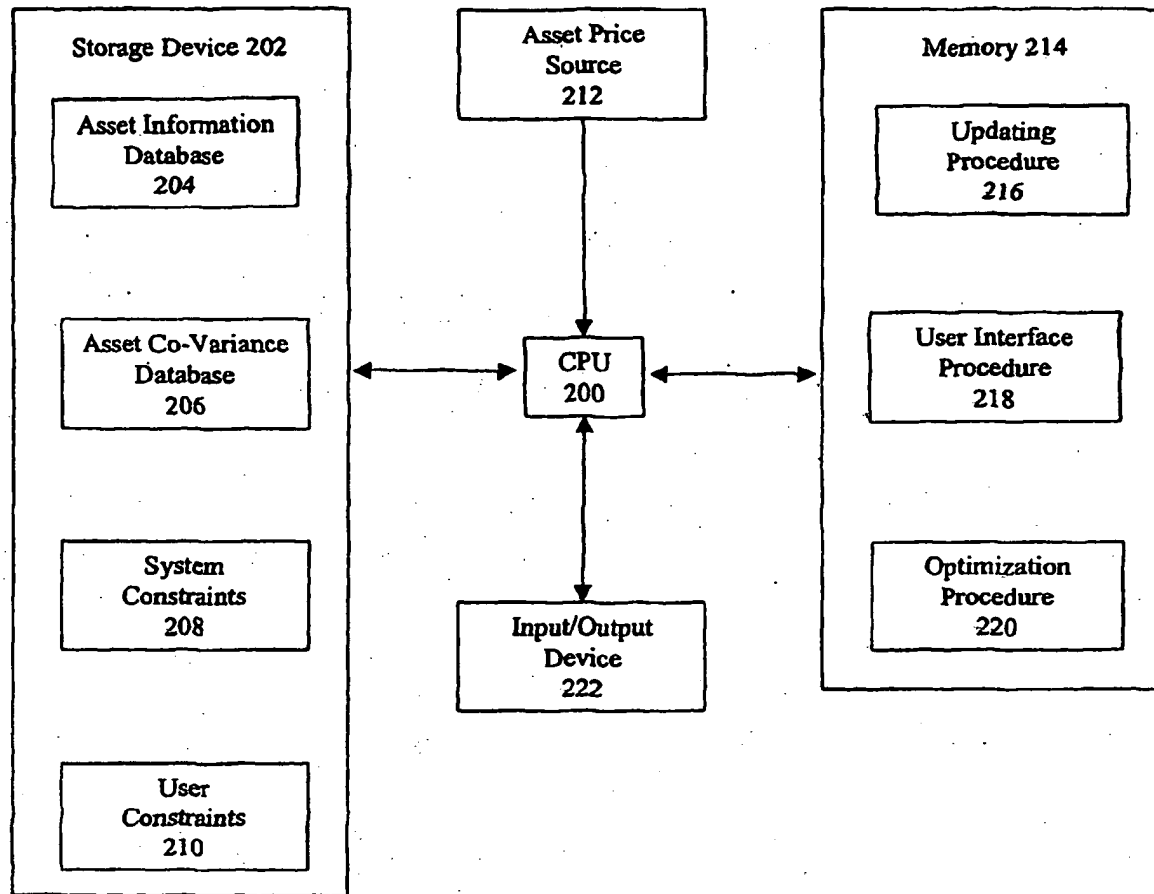
Fig. 2A

Fig. 2B

Fig. 3

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US01/15544

A. CLASSIFICATION OF SUBJECT MATTER

IPC(7) :G06F 17/60
US CL :705/35, 36

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. : 705/35, 36

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

SNT, WEST, EAST

search terms: invest, portfolio, asset, penalty, ...

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 5,784,696 A (MELNIKOFF) 21 July 1998, col. 7-22	1-116
A	US 5,819,237 A (GARMAN) 06 October 1998, abstract.	1-116
A	US 5,761,442 A (BARR et al) 02 June 1998, abstract.	1-116

☐ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

*	Special categories of cited documents:	"T"	later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"A"	document defining the general state of the art which is not considered to be of particular relevance		
"E"	earlier document published on or after the international filing date	"X"	document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
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"O"	document referring to an oral disclosure, use, exhibition or other means		
"P"	document published prior to the international filing date but later than the priority date claimed	"&"	document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
22 JUNE 2001	**2 6 JUL 2001**

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